Problem 70. Posed by K.B. Reid.

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An n-tournament T is *nearly triply regular* (abbreviated NTR) if there exist integers j_1, j_2, k, r satisfying $0 \leqslant j_1 < j_2 \leqslant k < r$ and

(1) $\deg^+(x) = r$,

(2) $\deg^+(x, y) = k$, and

(3) $\deg^+(x, y, z) = j_1$ or j_2,

for every triple of distinct vertices x, y, z in T. The notation $\deg^+(x, y)$, for example, means all the vertices dominated by both x and y.

The quadratic residue tournaments of orders 7 and 11 are examples of NTR tournaments (with parameters $(j_1, j_2, k, r) = (0, 1, 1, 3)$ and $(0, 1, 2, 5)$, respectively). The quadratic residue tournaments of orders 19, 23 and 31 are not NTR tournaments. Conditions (1) and (2) require T to be doubly regular of order $4k+3$, and hence require the existence of a skew-Hadamard matrix of order $4k+4$; moreover, $j_1 = j_2$ is prohibited in condition (3) as there exist no 'triply regular' tournaments (see [1]).

Does there exist an infinite family of NTR tournaments?

References

[1] E. Brown and K.B. Reid, Doubly regular tournaments are equivalent to skew-Hadamard matrices, J. Combin. Theory Ser. B 12 (1972) 332–338.

[2] M. Herzog and K.B. Reid, Regularity in tournaments, in Theory and Applications of Graphs, Proc. Internat. Conference at Kalamazoo, 1976, Lecture Notes in Math. 642 (1978) 442–453.